7 Appendix

Formulae for estimation of expected survival

Under the Ederer I method (Ederer et al. 1961), the cumulative expected survival from the date of diagnosis to the end of the ith interval is given by

$$_{1}p_{i}^{*} = \sum_{h=1}^{l_{1}} {}_{1}p_{i}^{*}(h)/l_{1},$$

where l_1 is the total number of patients alive at the start of follow-up and ${}_1p_i^*(h)$ is the expected probability of surviving to the end of the *i*th interval for a person in the general population, similar to the *h*th patient alive at the beginning of follow-up with respect to age, sex, and calendar time, given by

$$_{1}p_{i}^{*}(h) = \prod_{j=1}^{i} p_{j}^{*}(h).$$

Under the Ederer II method (Ederer and Heise 1959)

$$_{1}p_{i}^{*} = \prod_{j=1}^{i} p_{j2}^{*},$$

where

$$p_{j2}^* = \sum_{h=1}^{l_j} p_j^*(h) / l_j$$

is the average of the annual expected survival probabilities $p_j^*(h)$ of the patients alive at the start of the *j*th interval.

The expected survival proportion using the Hakulinen method (Hakulinen 1982) is derived as follows. Let k_j be the number of patients with a potential follow-up time which extends beyond the beginning of the *j*th interval. Let the first k_{ja} of these k_j patients have a potential follow-up time which extends past the end of the *j*th interval and the last k_{jb} be potential withdrawals during the *j*th interval. It follows that $k_1 = l_1$, $k_{j+1} = k_{ja}$, and $k_j = k_{ja} + k_{jb}$. We will use the notation K_{ja} to refer to the set of k_{ja} patients etc. and *h* to index the k_{ja} patients in the set K_{ja} . The expected number of patients alive and under observation at the beginning of the *j*th interval is given by:

$$l_{j}^{*} = \begin{cases} \sum_{h \in K_{j}} p_{j-1}^{*}(h) & \text{for } j \geq 2\\ l_{1} & \text{for } j = 1 \end{cases}$$

For the k_{jb} patients with potential follow-up times ending during the *j*th interval, it is assumed that each patient is at risk for half of the interval, so the expected probability of dying during the interval is given by $1 - \sqrt{p_i^*}$. The expected number of patients

24

P.W. Dickman, E. Coviello, and M. Hills

withdrawing alive during the jth interval is therefore given by:

$$w_j^* = \begin{cases} \sum_{h \in K_{jb}} p_{j-1}^*(h) \sqrt{p_j^*(h)} & \text{for } j \ge 2\\ \sum_{h \in K_{1b}} \sqrt{p_1^*(h)} & \text{for } j = 1 \end{cases}$$

The expected number of patients dying during the *j*th interval, among the k_{jb} patients with potential follow-up time ending during the same interval is given by:

$$\delta_j^* = \begin{cases} \sum_{h \in K_{jb}} p_{j-1}^*(h) [1 - \sqrt{p_j^*(h)}] & \text{for } j \ge 2\\ \sum_{h \in K_{1b}} [1 - \sqrt{p_1^*(h)}] & \text{for } j = 1 \end{cases}$$

and the expected total number of patients dying during the jth interval is given by:

$$d_j^* = \begin{cases} \left\{ \sum_{h \in K_{ja}} p_{j-1}^*(h) [1 - p_j^*(h)] \right\} + \delta_j^* & \text{for } j \ge 2\\ \left\{ \sum_{h \in K_{1a}} [1 - p_1^*(h)] \right\} + \delta_1^* & \text{for } j = 1 \end{cases}$$

The expected interval-specific survival proportion is then written as:

$$g_j^* = 1 - d_j^* / (l_j^* - w_j^*/2),$$

and, finally, the expected survival proportion from the beginning of follow-up (usually diagnosis) to the end of the ith interval is obtained by calculating:

$$_1p_i^* = \prod_{j=1}^i g_j^*.$$